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REPORT DOCUMENTATION PAGE

AD-A232 701

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S) ARO 24092.15-MA
6a. NAME OF PERFORMING ORGANIZATION University of Wisconsin-Madison 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION U. S. Army Research Office
6c. ADDRESS (City, State, and ZIP Code) Center for the Mathematical Sciences 610 Walnut Street-11th Floor Madison, WI 53705	7b. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211
8a. NAME OF FUNDING/SPONSORING ORGANIZATION U. S. Army Research Office 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER DAAL03-87-K-0043
8c. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)

Some Problems in Nonlinear Analysis

12. PERSONAL AUTHOR(S)

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13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM 05/23/87 TO 05/22/90	December 17, 1990	5

16. SUPPLEMENTARY NOTATION The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
nonlinear semigroup theory, porous medium, Stefan problem, Hamilton-Jacobi equations, dynamic programming, differential games, viscosity solutions, calculus of variations, periodic solutions, Hamiltonian systems, singular potentials, (over)

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

M. G. Crandall has worked on several questions including applications of nonlinear semigroup theory to nonlinear diffusion problems, the abstract theory of evolution equations, the theoretical basis for Hamilton-Jacobi equations in infinite dimensions and its application to dynamic programming in infinite dimensional control and differential games, and the theory of viscosity solutions of fully nonlinear second order partial differential equations.

P. H. Rabinowitz has worked on a variety of problems which have the common feature that they all involve the development of methods in the calculus of variations and their application to differential equations. In particular he treated the existence of periodic solutions of smooth Hamiltonian systems and systems involving singular potentials, the existence of various types of connecting orbits of Hamiltonian systems such as homoclinic and heteroclinic solutions, and the existence of multiple solutions of semilinear elliptic equations on $\mathbb{R}^n$.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

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18. (continued) connecting orbits, homoclinic and heteroclinic solutions.

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SOME PROBLEMS IN NONLINEAR ANALYSIS

FINAL REPORT

Michael G. Crandall and Paul H. Rabinowitz

December 17, 1990

U. S. Army Research Office

DAAL03-87-K-0043

University of Wisconsin-Madison

Research of M. G. Crandall

The papers listed below treat several distinct areas of research.

The paper [1] is an application of the nonlinear semigroup theory to an archetypal nonlinear diffusion problem under nonlinear boundary conditions. The pde involved is a general form containing, for example, models of flow in a porous medium and the Stefan problem as special cases. The results establish existence and uniqueness of solutions of an associated semilinear elliptic problem and apply them, via the semigroup theory, to the diffusion problem. The study shows that necessary conditions for existence are also sufficient and that the solution depends continuously on the nonlinearities in the equation and the boundary conditions. A consequence of the study is that problems which are at best awkward to formulate classically can be approximated by problems with classical solutions in a continuous way.

Papers [2] and [9] are contributions to the abstract theory of evolution problems. Paper [2] completes a program to subsume some quasilinear evolution problems, previously studied by Kato, under the nonlinear semigroup framework, allowing time-dependence in the equation and the norm in which the solution should be measured to depend on the solution and the time, as is required to cover important symmetric hyperbolic systems. The results provide, from some points of view, a more congenial avenue to the quasilinear theory – which covers many important physical models – and show that the straightforward difference approximations used in the proof not only converge, but provide a natural basis to study existence, etc. The paper [9], although a proceedings papers, is largely original. It provides an abstract functional analytic framework for the discussion of evolutions generated by operators like the Laplacian or, more generally, the p-Laplacian, which behave well in almost all spaces. In the process, a simple outline of some interpolation theory is given, with applications showing that classical results for linear mappings may be obtained in a simple way even in the nonlinear case and new generation and perturbation theorems are obtained.

Papers [6] and [8] are a continuation of the authors' program to provide a theoretical basis for Hamilton-Jacobi equations in infinite dimensions, with an eye to developing a theoretical foundation for dynamic programming in infinite dimensional control and differential games. These works were the first to establish existence, uniqueness, etc., under conditions allowing the dynamics to include partial differential equations of evolution. Other workers are now entering this rich arena; we mention in particular D. Tataru who was stimulated by this series to make some dramatic recent contributions.

The paper [7] perfects a result [4] which allowed a significant simplification of the presentation of the theory of viscosity solutions of fully nonlinear second order partial differential equations. This theory is now an enormously active area, finding new applications in asymptotics, geometry, control, etc. almost daily. Using the results as formulated in [7], [11] is a primarily (but not completely) expository presentation of the theory. Indeed, we expect [11] to have a very substantial impact as it simplifies many aspects of working with viscosity solutions and presents the main ideas and results in a congenial and organized way for the first time. Preliminary response indicates that it will be widely read. It will appear in the Bulletin of the Americal Mathematical Society as a "research expository article". The paper [10] is a related short proceedings article intended to advertise these developments to its audience.

Publications of M. G. Crandall

[1] Ph. Benilan, M. G. Crandall and P. Sacks, Some L^1 Existence and Dependence Results for Semilinear Elliptic Equations under Nonlinear Boundary Conditions, *Appl. Math. Optim.* **17** (1988), pp. 203-224.

[2] M. G. Crandall and P. E. Souganidis, On Nonlinear Equations of Evolution, *Nonlin. Anal. Th. Meth. Appl.* **13** (1989), pp. 1375-1392.

[3] M. G. Crandall, R. Newcomb and Y. Tomita, Existence and uniqueness of viscosity solutions of degenerate quasilinear elliptic equations in $\mathbb{R}^n$, *Applicable Analysis* **34** (1989), pp. 1-23.

[4] M. G. Crandall, Quadratic forms, semidifferentials and viscosity solutions of fully nonlinear elliptic equations, *Ann. I.H.P. Anal. Non. Lin.* **6** (1989), pp. 419-435.

[5] M. G. Crandall and P.-L. Lions, Quadratic growth of solutions of fully nonlinear second order equations on $\mathbb{R}^n$, *Diff. Int. Equa.* **3** (1990), pp. 601-616.

[6] M. G. Crandall and P.-L. Lions, Hamilton-Jacobi equations in infinite dimensions: Part IV. Unbounded linear terms, *J. Func. Anal.* **90** (1990), pp. 237-283.

[7] M. G. Crandall and H. Ishii, The maximum principle for semicontinuous functions, *Diff. and Int. Equations* **3** (1990), pp. 1001-1014.

[8] M. G. Crandall and P.-L. Lions, Hamilton-Jacobi equations in infinite dimensions: Part V. B-continuous solutions, *J. Func. Anal.*, in press.

[9] Ph. Benilan and M. G. Crandall, Completely Accretive Operators, Proceedings of the International Symposium on Semigroup Theory and Partial Differential Equations, Delft, Netherlands, 1989, Marcel Dekker, in press.

[10] M. G. Crandall, The Maximum principle, semicontinuity and nonlinear pde's, Proceedings of the 29th IEEE Conference on Decision and Control, Honolulu, 1990.

[11] M. G. Crandall, H. Ishii and P.-L. Lions, Users guide to viscosity solutions of second order partial differential equations, *Bull. Amer. Math. Soc.*, to appear.

Research of P. H. Rabinowitz

Rabinowitz's research during the period of this grant has resulted in 13 papers. Paper [1] studies Hamiltonian systems of compound pendulum type, i.e. the nonlinear terms are periodic in both the independent and dependent variables. It combines the resulting $\mathbf{Z}^n$ symmetry with variational arguments to prove the existence of multiple time periodic solutions.

Aside from the last paper [13], the problems studied in the remaining papers fall into two classes: (a) periodic solutions of singular Hamiltonian systems and (b) connecting orbits of Hamiltonian systems, especially heteroclinic and homoclinic solutions. Papers [2], [4–6], and [9] deal with (a). In [2] and [4], partly jointly with A. Bahri, Rabinowitz found new minimax methods to prove the existence of infinitely many periodic solutions of autonomous and forced singular Hamiltonian systems. The type of singularity treated was a point singularity such as arises in the Kepler problem. E.g. a simple model case is

$$\ddot{q} + V_q(t, q) = 0 \tag{1}$$

where $q \in \mathbb{R}^n$, $V \in C^1(\mathbb{R}^n, \mathbb{R})$, and V behaves like $-|q|^{-\beta}$ for q near 0 and $\beta > 0$. In [5–6] and [9], joint with Bahri, Rabinowitz devised new indirect variational methods which establish the existence of infinitely many periodic solutions of problems of 3-body type. Here a model case is

$$m_i \ddot{q}_i + \frac{\partial V}{\partial q_i}(q_1, q_2, q_3) = 0, \qquad i = 1, 2, 3 \tag{2}$$

$m_i > 0$, $q \in \mathbb{R}^\ell$, and

$$V(q) = -\sum_{\substack{i,j=1 \\ i \neq j}}^{3} \frac{\alpha_{ij}}{|q_i - q_j|^{\beta_{ij}}}$$

with α_{ij}, $\beta_{ij} > 0$.

Papers [3], [7–8], [10–12] deal with connecting orbits of Hamiltonian systems. In [3], [10], [11], the existence of heteroclinic orbits was studied for a subclass of the potentials treated in [1]. Novel minimization arguments were used to find such orbits as well as heteroclinic chains. In [7], joint with Tanaka, some of the ideas from [3] were extended to study orbits emanating from local or global maxima of the potential energy. Paper [8] uses the Mountain Pass Theorem and approximation arguments to establish the existence of a homoclinic orbit as a limit of subharmonic solutions for a class of Hamiltonian systems having a superquadratic potential. As a followup to this paper, in [12], jointly with Coti-Zelati, a rather novel variational approach was discovered which establishes the existence of multiple homoclinic solutions for Hamiltonian systems in the setting of [8]. In fact, infinitely many homoclinics were obtained near each level set of the corresponding functional for each integer multiple of the mountain pass critical value.

Finally in [13], some of the ideas from [8] were used to get the existence of positive solutions for a class of semilinear elliptic partial differential equations on $\mathbb{R}^n$.

Publications of P. H. Rabinowitz

1. On a class of functionals invariant under a $\mathbf{Z}^n$ action, Trans. Amer. Math. Soc. **310** (1988), 303–311.
2. A minimax method for a class of functionals with singular potentials (with A. Bahri), J. Functional Analysis **82** (1989), 412–428.
3. Periodic and heteroclinic orbits for a periodic Hamiltonian system, Analyse Nonlineaire **6** (1989), 331–346.
4. Periodic solutions of some forced singular Hamiltonian systems, Analysis, Et Cetera (P. H. Rabinowitz and E. Zehnder, eds.), Academic Press, 1990, 521–544.
5. Orbites periodiques des systèmes hamiltoniens singular du type de celui des trois corps (with A. Bahri), C. R. Acad. Sci. **310** ser. I (1990), 155–160.
6. Periodic solutions of Hamiltonian systems of 3-body type (with A. Bahri), to appear Analyse Nonlineaire.
7. Some results on connecting orbits for a class of Hamiltonian systems (with K. Tanaka), to appear Math. Z.
8. Homoclinic orbits for a class of Hamiltonian systems, Proc. Roy. Soc. Edinburgh **114A** (1990), 33–38.
9. Periodic solutions of some problems of 3-body type (with A. Bahri), to appear Proc. Int. Conf. on Variational Problems, Paris, July 1989.
10. A variational approach to heteroclinic orbits for a class of Hamiltonian systems, to appear, Festschrift in honor of J. L. Lions.
11. Some recent results on heteroclinic and other connecting orbits of Hamiltonian systems, to appear, Proc. Int. Conf. on Nonlinear Analysis and Variational Methods, L'Aquilá, Jan. 1990.
12. Homoclinic orbits for second order Hamiltonian systems possessing superquadratic potentials (with V. Coti-Zelati), submitted to J. Amer. Math. Soc.
13. A note on a semilinear elliptic equation on $\mathbb{R}^n$, to appear in Nonlinear Analysis, a tribute in honor of Giovanni Prodi, Quaderni Scuola Norm. Sup. Pisa.